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HIGHLY INTENSIVE HEAT AND MASS TRANSFER IN DISPERSED MEDIA

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Abstract—A solution is given for a system of differential equations for heat and mass transfer in a dispersed medium in the presence of phase conversions (the evaporation of liquid or steam condensation). The mass transfer is thought to occur under the effect of mass transfer potential gradient $\nabla\theta$, a temperature gradient ∇t and a total pressure gradient ∇p. Solutions are obtained for the heat and mass transfer potentials (t, θ, p) applicable to a one-dimensional problem (an infinite plate and a sphere with boundary conditions of the third order. The solutions are given in the criterion form using heat and mass transfer similarity criteria. An analysis is given of the effect of the separate similarity criteria (Bi_q, Bi_m, Pn, Lu_p) on the fields of heat and mass transfer potentials. The effect of molar (filtration) transfer on heat and mass transfer is shown.

Résumé—La solution du système différentiel régissant le transfert de chaleur et de masse dans un milieu dispersé a été donné en présence de changement de phase (évaporation du liquide et condensation de vapeur). Le transfert de masse est supposé se produire sous l'effet du gradient de potentiel de transfert de masse $\nabla\theta$, du gradient de température ∇t et du gradient de pression totale ∇p. Les solutions pour les potentiels de transfert de chaleur et de masse (t, θ, p) sont obtenues dans le cas du problème unidimensionnel (plaque infinie et sphère) et avec des conditions aux limites de 3ème espèce. Les solutions sont données sous forme de critère par application des critères de similarité relatifs au transfert de chaleur et de masse. Une analyse a été faite sur l'influence des critères de similarité particuliers (Bi_q, Bi_m, Pn, Lu_p) sur champs de potentiel de transfert de chaleur et de masse.

Zusammenfassung—Die Differentialgleichungen für Wärme- und Stoffübertragung im dispersen Medium wurden bei gleichzeitiger Phasenumwandlung gelöst (Flüssigkeitsverdampfung oder Dampfkondensation). Es wurde angenommen, dass die Stoffübertragung stattfindet unter der gleichzeitigen Wirkung des Potentialgradienten der Stoffübertragung $\nabla\theta$, dem Temperaturgradienten ∇t und dem Gradienten des Gesamtdruckes ∇p. Die Lösungen für die Potentiale der Wärme- und Stoffübertragung (t, θ, p) werden angegeben für das eindimensionale Problem (unendliche Platte und Kugel) bei Randbedingungen dritter Art. Die Lösungen sind in Form von Kenngrössen mitgeteilt. Der Einfluss der besonderen Kenngrössen (Bi_q, Bi_m, Pn, Lu_p) auf die Potentialfelder der Wärme- und Stoffübertragung wurde untersucht.

Abstract—В статье дано решение системы дифференциальных уравнений тепло- и массопереноса в дисперсной среде при наличии фазовых превращений (испарение жидкости или конденсация пара). Предполагается, что перенос массы вещества происходит под действием градиента потенциала массопереноса $\nabla\theta$, градиента температуры ∇t и градиента общего давления. ∇p. Получены решения для потенциалов переноса тепла и массы вещества (t, θ, p) применительно к одномерной задаче (неограниченная пластина и шар) при граничных условиях третьего рода. Решения даны в критериальной форме с использованием критериев подобия тепломассопереноса. Дан анализ влияния отдельных критериев подобия (Bi_q, Bi_m, Pn, Lu_p) на поля потенциалов тепломассопереноса. Показано влияние молярного (фильтрационного) переноса массы на тепло- и массоперенос.

IN order to intensify heat and mass transfer high temperatures and pressures are employed in modern engineering on an increasing scale. Under such conditions the mechanism of transfer of matter and energy is considerably modified. In addition to transfer due to molecular processes, molar processes of a filtrational nature start playing an important part. Let us consider highly intensive internal heat and mass transfer in moist dispersed media, and the influence of various factors on such transfer.

The transfer of heat and that of matter in dispersed media are interconnected and interdependent. It is being effected by the action of various thermodynamic forces: heat transfer, mass transfer and filtration. On the other hand, the magnitude of the thermodynamic force is determined by the difference of the respective transfer potential. The heat transfer potential — t the potential of molecular mass transfer θ, and the potential of filtration (molar) transfer— the general or excess pressure p [1]. The system of differential equations for heat and mass exchange in the case of molecular and molar transfer can be represented as follows [2]:

$$c_q\gamma \frac{\partial t}{\partial \tau} = \nabla(\lambda_q \nabla t) + \epsilon\rho\gamma\, c_m \frac{\partial \theta}{\partial_\tau} - \sum_i c_i q_{mi} \nabla t$$

$$c_m\gamma \frac{\partial \theta}{\partial \tau} = \nabla[\lambda_m(\nabla\theta + \delta\nabla t) + \lambda_p \nabla p] \qquad (1)$$

$$c_p\gamma \frac{\partial p}{\partial \tau} = \nabla(\lambda_p \nabla p) - \epsilon\gamma\, c_m \frac{\partial \theta}{\partial \tau}$$

where λ_q is the coefficient of heat conductivity; λ_m and λ_p are the coefficients of molecular filtration conductivity of matter; c_q, c_m and c_p are the respective heat, liquid and vapour capacities of the material; ρ is the specific heat of phase transition; δ is the thermogradient coefficient; a_m is the mass transfer coefficient of potential conductivity; γ is the density of the dispersed medium; ϵ is the criterion of phase transition; τ is the time; the added term $\sum_i c_i q_{im} \nabla t$ characterizes the convective component of heat transfer in the dispersed medium.

In order to connect the dispersed medium with the surrounding medium we assume boundary conditions of the third kind, since they represent a mathematical analogy of frequently occurring convection processes. In case of highly intensive heat and mass transfer they can be expressed as follows:

$$-\lambda_q(\nabla t)_s + \alpha_q[t_c - (t)_s] - (1 - \epsilon)\rho\alpha_m[(\theta)_s - \theta_e] = 0,$$

$$\lambda_m(\nabla\theta)_s + \lambda_m\delta(\nabla t)_s + \lambda_p(\nabla p)_s + \alpha_m[(\theta)_s - \theta_e] = 0, \qquad (2)$$

$$(p)_s = p_0,$$

where α_q and α_m, respectively, denote the coefficients of heat exchange and mass transfer between the carrier of heat and the dispersed medium; indices s refer to the surface of the dispersed medium, and indices c to the surrounding medium; index 0 refers to the initial value of the respective parameter, and index e refers to its equilibrium value.

The transfer coefficients and the thermodynamic characteristics of the disperse medium are in one way or other functions of co-ordinates and time. In the first approximation the coordinate relation can be neglected. The system of equations (1) is thus simplified and permits evaluation of the process for each zone [3, 4].

The solution of the system of equations (1) for one-dimensional bodies under boundary conditions (2) and with constant initial distribution of transfer potentials is as follows: for an infinite plate

$$T(X,\mathrm{Fo}) = 1 - \sum_{n=1}^{\infty} \sum_{j=1}^{3} C_{nj} \cos \nu_j \mu_n X \cdot \exp(-\mu_n^2 \mathrm{Fo})$$

$$\Theta(X,\mathrm{Fo}) = 1 + \frac{1}{\epsilon \mathrm{Ko}} \sum_{n=1}^{\infty} \sum_{j=1}^{3} C_{nj} (1 - \nu_j^2) \cos \nu_j \mu_n X \cdot \exp(-\mu_n^2 \mathrm{Fo})$$

$$P(X,\mathrm{Fo}) = -\frac{1}{\epsilon \mathrm{Bu}} \sum_{n=1}^{\infty} \sum_{j=1}^{3} C_{nj}\sigma_j \cos \nu_j \mu_n X \cdot \exp(-\mu_n^2 \mathrm{Fo})$$

for a sphere

$$T(X,\mathrm{Fo}) = 1 + \sum_{n=1}^{\infty}\sum_{j=1}^{3} C_{nj}\frac{\sin \nu_j\mu_n X}{X}\cdot \exp(-\mu_n^2\mathrm{Fo})$$

$$\Theta(X,\mathrm{Fo}) = 1 - \frac{1}{\epsilon\mathrm{Ko}}\sum_{n=1}^{\infty}\sum_{j=1}^{3} C_{nj}(1-\nu_j^2)\frac{\sin \nu_j\mu_n X}{X}\cdot \exp(-\mu_n^2\mathrm{Fo})$$

$$P(X,\mathrm{Fo}) = \frac{1}{\epsilon\mathrm{Bu}}\sum_{n=1}^{\infty}\sum_{j=1}^{3} C_{nj}\sigma_j\frac{\sin \nu_j\mu_n X}{X}\cdot \exp(-\mu_n^2\mathrm{Fo})$$

where

$$C_{n1} = 2\,\frac{(\epsilon\mathrm{Ko}\,k_1 - 1)(P_{n2} - (X_{n2}/X_{n3})\,P_{n3}) - \epsilon\mathrm{Ko}(Q_{n2} - (X_{n2}/X_{n3})\,Q_{n3})}{\mu_n\psi_n}$$

$$C_{n2} = -2\,\frac{(\epsilon\mathrm{Ko}k_1 - 1)(P_{n1} - (X_{n1}/X_{n3})\,P_{n3}) - \epsilon\mathrm{Ko}(Q_{n1} - (X_{n1}/X_{n3})\,Q_{n3})}{\mu_n\psi_n}$$

$$C_{n3} = 2\,\frac{(\epsilon\mathrm{Ko}k_1 - 1)(P_{n1}\,(X_{n2}/X_{n3}) - P_{n2}\,(X_{n1}/X_{n3}) - \epsilon\mathrm{Ko}(Q_{n1}\,(X_{n2}/X_{n3}) - Q_{n2}\,(X_{n1}/X_{n3})}{\mu_n\psi_n}$$

$$\psi_n = \left(P_{n1} - \frac{X_{n1}}{X_{n3}}P_{n3}\right)\left[\nu_2 A_{n2} - \frac{X_{n2}}{X_{n3}}(\nu_3 A_{n3} + b_{n2}Q_{n3})\right] + \left(Q_{n2} - \frac{X_{n2}}{X_{n3}}Q_{n3}\right)\left[\nu_1 B_{n1} - \frac{X_{n1}}{X_{n3}}(\nu_3 B_{n3} + b_{n1}P_{n3})\right] - \left(P_{n2} - \frac{X_{n2}}{X_{n3}}P_{n3}\right)\left[\nu_1 A_{n1} - \frac{X_{n1}}{X_{n3}}(\nu_3 A_{n3} + b_{n1}Q_{n3})\right] - \left(Q_{n1} - \frac{X_{n1}}{X_{n3}}Q_{n3}\right)\left[\nu_2 B_{n2} - \frac{X_{n2}}{X_{n3}}(\nu_3 B_{n3} + b_{n2}P_{n3})\right]$$

$$\sigma_j = (1-\epsilon)(1-\nu_j^2) - \mathrm{Lu}(1-\nu_j^2)\,\nu_j^2 - \epsilon\mathrm{KoPnLu}\,\nu_j^2$$

$$\nu_j = +\sqrt{(\Lambda_j + \tfrac{1}{3}\beta_1)},\ (j = 1, 2, 3)$$

$$\Lambda = \sqrt[3]{\{-\tfrac{1}{2}\Pi_2 + \sqrt{(\tfrac{1}{4}\Pi_2^2 + \tfrac{1}{27}\Pi_1^3)}\}} + \sqrt[3]{\{-\tfrac{1}{2}\Pi_2 - \sqrt{(\tfrac{1}{4}\Pi_2^2 + \tfrac{1}{27}\Pi_1^3)}\}}$$

Here the correct values of Λ_j must simultaneously satisfy the following relation:

$$\sqrt[3]{\{-\tfrac{1}{2}\Pi_2 + \sqrt{(\tfrac{1}{4}\Pi_2^2 + \tfrac{1}{27}\Pi_1^3)}\}}\cdot\sqrt[3]{\{-\tfrac{1}{2}\Pi_2 - \sqrt{(\tfrac{1}{4}\Pi_2^2 + \tfrac{1}{27}\Pi_1^3)}\}} = -\tfrac{1}{3}\Pi_1$$

where

$$\Pi_1 = -\frac{\beta_1^2}{3} + \beta_2,\ \Pi_2 = -\frac{2\beta_1^3}{27} + \frac{\beta_1\beta_2}{3} - \beta_3$$

$$\beta_1 = 1 + (1-\epsilon)\frac{1}{\mathrm{Lu}} + \frac{1}{\mathrm{Lu}_p} + \epsilon\mathrm{KoPn}$$

$$\beta_2 = (1-\epsilon)\frac{1}{\mathrm{Lu}} + \left(1 + \frac{1}{\mathrm{Lu}} + \epsilon\mathrm{KoPn}\right)\frac{1}{\mathrm{Lu}_p};\quad \beta_3 = \frac{1}{\mathrm{Lu}\cdot\mathrm{Lu}_p}$$

Also, for an infinite plate

$$A_{nj} = \left[1 + (1-\nu_j^2)\,K_1 + \frac{1}{\mathrm{Bi}_q}\right]\sin\nu_j\mu_n + \frac{\nu_j\mu_n}{\mathrm{Bi}_q}\cos\nu_j\mu_n$$

$$B_{nj} = (1 - \nu_j^2) \sin \nu_j\mu_n + \frac{(1 - \nu_j^2) + \epsilon\mathrm{KoPn} + \sigma_j \mathrm{Lu}_p/\mathrm{Lu}}{\mathrm{Bi}_m} (\sin \nu_j\mu_n + \nu_j\mu_n \cos \nu_j\mu_n)$$

$$P_{nj} = (1 - \nu_j^2) \cos \nu_j\mu_n - \frac{(1 - \nu_j^2) + \epsilon\mathrm{KoPn} + \sigma_j \mathrm{Lu}_p/\mathrm{Lu}}{\mathrm{Bi}_m} \nu_j\mu_n \sin \nu_j\mu_n$$

$$Q_{nj} = [1 + (1 - \nu_j^2) K_1] \cos \nu_j\mu_n - \frac{\nu_j\mu_n}{\mathrm{Bi}_q} \sin \nu_j\mu_n$$

$$b_{nj} = \nu_j \tan \nu_j\mu_n - \nu_3 \tan \nu_3\mu_n, \quad X_{nj} = \sigma_j \cos \nu_j\mu_n$$

for a sphere

$$A_{nj} = [1 + (1 - \nu_j^2) K_1] \cos \nu_j\mu_n - \frac{\nu_j\mu_n}{\mathrm{Bi}_q} \sin \nu_j\mu_n$$

$$B_{nj} = (1 - \nu_j^2) \cos \nu_j\mu_n - \frac{(1 - \nu_j^2) + \epsilon\mathrm{KoPn} + \sigma_j \mathrm{Lu}_p/\mathrm{Lu}}{\mathrm{Bi}_m} \nu_j\mu_n \sin \nu_j\mu_n$$

$$P_{nj} = (1 - \nu_j^2) \sin \nu_j\mu_n + \frac{(1 - \nu_j^2) + \epsilon\mathrm{KoPn} + \sigma_j \mathrm{Lu}_p/\mathrm{Lu}}{\mathrm{Bi}_m} (\nu_j\mu_n \cos \nu_j\mu_n - \sin \nu_j\mu_n)$$

$$Q_{nj} = \frac{1}{\mathrm{Bi}_q} \nu_j\mu_n \cos \nu_j\mu_n + \left[1 - \frac{1}{\mathrm{Bi}_q} + (1 - \nu_j^2) K_1\right] \cdot \sin \nu_j\mu_n$$

$$b_{nj} = \nu_j \cot \nu_j\mu_n - \nu_3 \cot \nu_3\mu_n, \quad X_{nj} = \sigma_j \sin \nu_j\mu_n$$

μ_n is determined by the solutions of the characteristic equation

$$\left(P_{n1} - \frac{X_{n1}}{X_{n3}} P_{n3}\right)\left(Q_{n2} - \frac{X_{n2}}{X_{n3}} Q_{n3}\right) - \left(P_{n2} - \frac{X_{n2}}{X_{n3}} P_{n3}\right)\left(Q_{n1} - \frac{X_{n1}}{X_{n3}} Q_{n3}\right) = 0.$$

The values of the characteristic roots μ_1 for a dispersed medium possessing the form of an infinite plate are given in Table 1.

The solutions are given in dimensionless form: T, Θ, P denote, respectively, the dimensionless values of temperature, molecular and molar potentials, viz.

$$T = \frac{t - t_0}{t_c - t_0}, \quad \Theta = \frac{\theta_0 - \theta}{\theta_0 - \theta_e}, \quad P = \frac{p - p_0}{p_0}.$$

where

$X = \dfrac{x}{R}$, dimensionless co-ordinate;

$\mathrm{Fo} = \dfrac{a_q\tau}{R^2}$, criterion of homochronism of temperature fields, the so-called Fourier criterion (a_q is the heat coefficient of potential transfer $a_q = \lambda_q/c_q\gamma$);

$\mathrm{Lu} = \dfrac{a_m}{a_q}$, criterion of inertia of fields of heat and mass content, the so-called Luikov criterion, a_m is the liquid coefficient of potential transfer $a_m = \lambda_m/c_m\gamma$;

$\mathrm{Lu}_p = \dfrac{a_p}{a_q}$, criterion of inertia of filtration potential field with respect to temperature field (a_p is the filtrational coefficient of potential transfer $a_p = \lambda_p/c_p\gamma$);

$$\mathrm{Bi}_q = \frac{a_q R}{\lambda_q}$$

and $\mathrm{Bi}_m = \dfrac{a_m R}{\lambda_m a_m}$, Biot criteria of heat and mass transfer;

$\mathrm{Pn} = \dfrac{\delta(t_c - t_0)}{\theta_0 - \theta_e}$, Posnov criterion;

$\mathrm{Ko} = \dfrac{c_m \rho(\theta_0 - \theta_e)}{c_q(t_c - t_0)}$ Kossovich criterion;

$\mathrm{Bu} = \dfrac{\rho c_p}{c_q} \dfrac{p_0}{t_c - t_0}$, Bulygin criterion;

$K_1 = \dfrac{1 - \epsilon}{\epsilon} \mathrm{Lu} \dfrac{\mathrm{Bi}_m}{\mathrm{Bi}_q}$, complex criterion.

An analysis of the solution shows that the infinite series which the expressions for T, Θ and P contain, converge very quickly with increase of Fo. Beginning with Fo $\approx$ 0·7 it is possible with an accuracy of 1 per cent to limit oneself to two or three terms of a series.

The solution of the system of differential equations (1) (2) gives the dependence of the process on a large number of similarity criteria of heat and mass transfer. It must be stressed, however, that not all of these criteria have equal effect on the development of the process. Some of them primarily affect heat transfer, others—mass transfer, others again—filtrational characteristics of the transfer. Some of the criteria are connected with molecular, others with molar transfer. Let us consider the effect of various similarity criteria on highly intensive internal heat and mass transfer.

The heat and mass criteria of Biot

$$\mathrm{Bi}_q = \frac{a_q}{\lambda_q} R, \qquad \mathrm{Bi}_m = \frac{a_m}{\lambda_m} R$$

Table 1

Bi_q	Lu			Lu_p		ϵ		Bi_m			KoPn			
	0·3*	0·7	1·0	300	1000	0·1	0·9	5	10	15	0·45	1·50	3·00	5·85
0·30	0·469	0·0779	0·0578	0·454	0·468	0·463	0·474	0·454	0·466	0·468	0·508	0·485	0·456	0·412
0·35	0·495	0·0834	0·0622	0·480	0·493	0·490	0·501	0·476	0·492	0·494	0·544	0·513	0·480	0·430
0·40	0·517	0·0884	0·0662	0·503	0·517	0·512	0·523	0·493	0·513	0·515	0·574	0·538	0·501	0·445
0·45	0·536	0·0928	0·0699	0·523	0·537	0·533	0·543	0·511	0·532	0·534	0·602	0·563	0·518	0·458
0·50	0·556	0·0968	0·0733	0·541	0·554	0·550	0·561	0·528	0·552	0·553	0·625	0·582	0·534	0·470
0·60	0·583	0·1050	0·0801	0·572	0·584	0·579	0·590	0·555	0·577	0·581	0·669	0·617	0·560	0·488
0·70	0·607	0·111	0·0858	0·596	0·607	0·603	0·614	0·578	0·599	0·605	0·709	0·646	0·581	0·504
0·80	0·627	0·117	0·0911	0·617	0·628	0·623	0·633	0·596	0·617	0·624	0·741	0·670	0·599	0·515
0·90	0·645	0·123	0·0962	0·634	0·644	0·639	0·649	0·611	0·633	0·640	0·771	0·690	0·613	0·525
1·0	0·658	0·128	0·1011	0·648	0·658	0·653	0·662	0·622	0·645	0·653	0·796	0·707	0·625	0·533
1·5	0·706	0·151	0·1197	0·699	0·707	0·702	0·709	0·656	0·686	0·700	0·885	0·765	0·666	0·558
2·0	0·733	0·170	0·1340	0·727	0·734	0·731	0·736	0·676	0·714	0·725	0·938	0·798	0·690	0·572
3·0	0·764	0·197	0·157	0·758	0·765	0·761	0·767	0·696	0·739	0·755	0·993	0·836	0·716	0·589
4·0	0·779	0·214	0·173	0·775	0·781	0·776	0·784	0·704	0·752	0·771	1·023	0·856	0·730	0·599
5·0	0·789	0·228	0·184	0·786	0·791	0·785	0·794	0·712	0·761	0·781	1·042	0·867	0·738	0·604
6·0	0·796	0·242	0·197	0·793	0·799	0·792	0·803	0·715	0·767	0·787	1·054	0·875	0·744	0·608
7·0	0·801	0·250	0·208	0·798	0·803	0·796	0·807	0·718	0·771	0·792	1·061	0·881	0·748	0·611
8·0	0·804	0·254	0·214	0·802	0·807	0·800	0·812	0·721	0·775	0·796	1·068	0·886	0·751	0·613
9·0	0·807	0·260	0·220	0·805	0·810	0·802	0·815	0·723	0·777	0·798	1·073	0·889	0·754	0·614
10	0·810	0·263	0·225	0·808	0·813	0·804	0·817	0·725	0·779	0·801	1·077	0·891	0·756	0·616
15	0·817	0·280	0·245	0·816	0·820	0·810	0·825	0·728	0·786	0·808	1·090	0·900	0·761	0·621
20	0·820	0·287	0·256	0·819	0·824	0·813	0·828	0·730	0·788	0·812	1·096	0·904	0·764	0·622
30	0·824	0·296	0·269	0·823	0·828	0·816	0·832	0·732	0·792	0·816	1·103	0·908	0·766	0·624
40	0·826	0·300	0·275	0·825	0·830	0·818	0·834	0·734	0·794	0·817	1·105	0·911	0·769	0·626
50	0·827	0·302	0·278	0·826	0·831	0·819	0·836	0·735	0·795	0·818	1·106	0·913	0·770	0·627

* μ_1 values for Lu = 0·3 simulaneously correspond to μ_1 values for Lu_p = 500, ϵ = 0·7, Bi_m = 20, KoPn = 2·25

characterize the intensity of external transfer (Bi_q for heat and Bi_m for matter), as compared to the intensity of internal transfer. One might therefore call them criteria of surface heat and mass transfer. When the Biot coefficients are small the potentials in the centre of the material differ little from the corresponding potentials on its surface. Transfer of matter and heat is slow. With increase of Bi_q and Bi_m transfer intensifies. The surface temperature rapidly approaches that of the heat carrier. In a disperse medium big temperature gradients arise effecting intensive redistribution of matter. At the same time large stresses occur inside the material, creating conditions for warping, and formation of cracks and pores.

Once the processes within the material have become quasi-regular, i.e. once the Fourier criterion has reached the value of $\mathrm{Fo} = 0{\cdot}7 - 1{\cdot}0$, the Biot heat transfer criterion affects only the fields of the thermal characteristic (Fig. 1), and the Biot mass exchange criterion affects only the fields of molecular transfer potential or mass content (Fig. 2). The field of filtration potential becomes an automodelous one with respect to both criteria. A similar result has been found by considering molecular heat and mass transfer in moist material [5, 6]. This, as well as the fact that the Biot criteria are indifferent to the field of filtration potential indicates that the criteria of heat and mass exchange are chiefly connected with the molecular mechanism of transfer. The nature of the change of criteria with molecular transfer potential and temperature is in good accordance with experimental results obtained by Lebedev [7].

The criteria of Posnov

$$\mathrm{Pn} = \frac{\delta(t_c - t_0)}{\theta_0 - \theta_e},$$

of Kossovich

$$\mathrm{Ko} = \frac{c_m \rho(\theta_0 - \theta_e)}{c_q(t_c - t_0)},$$

of phase change (ϵ), and of Bulygin

$$\mathrm{Bu} = \frac{\rho c_p}{c_q} \frac{p_0}{t_c - t_0}$$

can be considered as belonging to the group of criteria of internal heat and mass transfer because they characterize deeper processes of phase change and transfer taking place within the material.

The Posnov criterion characterizes the relative non-uniformity of the mass content field produced by heat transfer. The Kossovich criterion is a specific form of the phase change criterion. It is equal to the ratio between specific heat consumed on evaporation of matter and specific heat consumed on heating the dispersed medium.

If there is no molar transfer in the material the nature of the effect of the criteria Pn and Ko on the potential of heat and mass transfer is

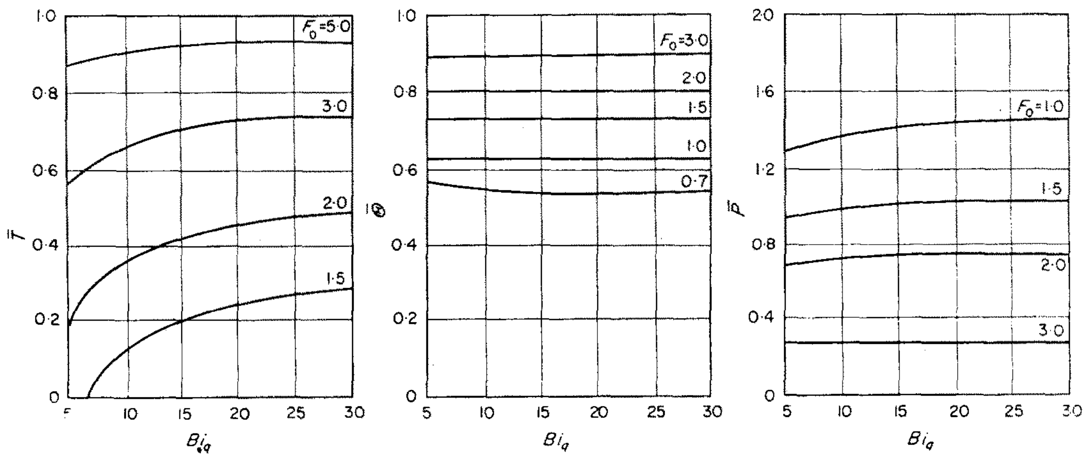

FIG. 1. Dimensionless transfer potentials as a function of Bi_q criterion

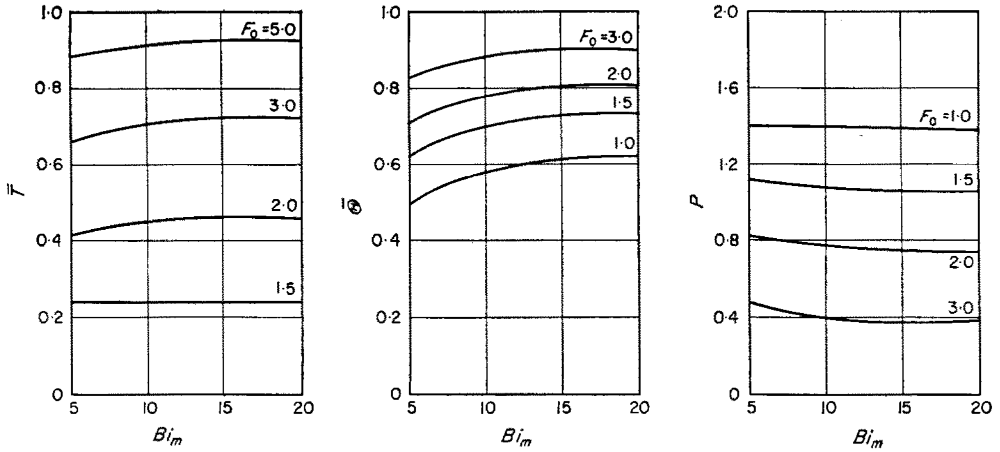

FIG. 2. Dimensionless transfer potentials as a function of Bi_m criterion

analogous to the effect on these potentials of the criteria of surface heat and mass transfer Bi_q and Bi_m. Under conditions of quasi-regular process the criterion Pn is automodelous with respect to the temperature field whilst the criterion Ko is automodelous with respect to the field of potential of molecular transfer of matter [5, 6]. The fact that molar transfer of matter starts taking place in the material changes the dependence of dimensionless potentials on the criteria Pn and Ko. At small values of Fo the temperature of the material increases visibly with increase of Pn, but in the course of time this effect diminishes and changes its sign at values of the Fourier criterion larger than two (Fig. 3). The dependence of the filtration potential on Pn, combined with the new nature of the influence on the temperature distribution shows that the Posnov criterion is connected with a molecular as well as with a molar transfer mechanism. A similar conclusion can be obtained with respect to the Kossovich criterion.

The fact that a powerful molar mechanism arises in the material has its effect first of all on redistribution of matter. The Posnov criterion which in case of molecular transfer characterizes internal processes of mass exchange must show

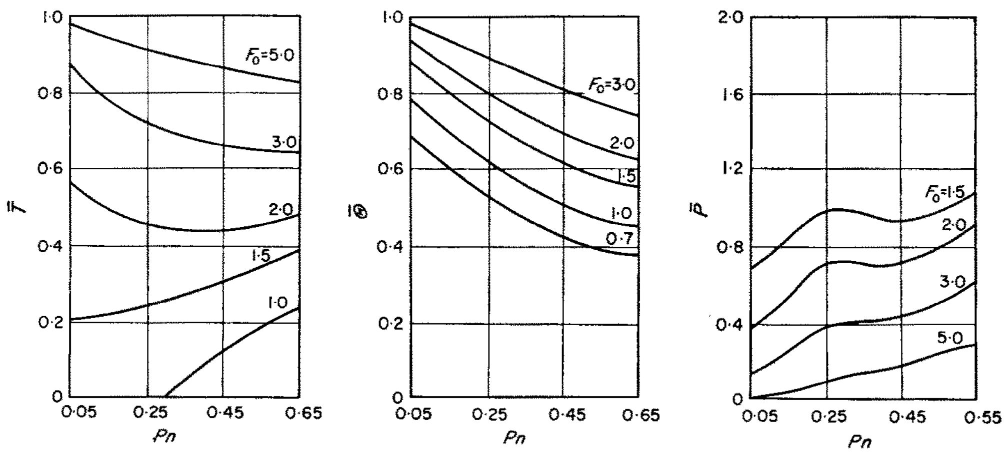

FIG. 3. Dimensionless transfer potentials as a function of Pn criterion

the appearance of a new mechanism much more clearly than the Kossovich criterion which characterizes internal processes of heat exchange. An analysis of the results of solution of the system of equations confirms this: the change of the effect of criterion Pn on the index of heat transfer of the process proves to be stronger than the change of criterion Ko with respect to the index of mass transfer.

With increase of criterion Pn the dimensionless potential of transfer of matter diminishes. This indicates that within equal intervals of time at smaller values of the Posnov criterion larger quantities of matter are being removed from the material. The effect of criterion Ko manifests itself primarily in the temperature field where the above-mentioned similarity between the Kossovich criterion and the Biot heat exchange criterion remains.

The criterion of phase change (ϵ) has the same effect on the internal process of heat and mass transfer as the Kossovich criterion. The difference lies only in the intensity of effect on the thermal characteristics: the effect of ϵ is weaker than that of Ko.

New and specific for high temperature heat and mass exchange is the Bulygin criterion Bu. Like the Kossovich criterion, the Bulygin criterion characterizes the accumulating power of the body. However, as distinct from the former, it concerns not the total heat consumed on formation of vapour but only the heat consumed on formation of vapour participating in molar transfer. The Bulygin criterion is closely linked with molar transfer and therefore affects only excess pressure distribution in the material, and not the temperature distribution and the potential of molecular transfer. The criterion Bu changes over a very wide range. The magnitude of the mean value of the filtration potential in the material is inversely proportional to the criterion and diminishes rapidly with increase of the latter.

The most essential effect on transfer potential belongs to the Luikov criterion (Lu $= a_m/a_q$) which may also be called the criterion of correlation between heat and mass transfer. The Luikov criterion characterizes the inertia of the potential field of molecular transfer of matter in relation to the field of heat transfer potential. In case of purely molecular transfer mechanism the value Lu $\approx 1{\cdot}0$ (with an accuracy up to 12 per cent) forms the limit for symmetry of relative relaxation speed of the above-mentioned potentials. Thus, if the criterion Lu is smaller than unity the propagation of temperature in the material occurs at higher speed than that of the mass transfer potential; if Lu is larger than unity the reverse takes place. If intensive molar transfer arises within the material, the symmetry observed in case of molecular transfer disappears, although the general trend of the process remains unchanged. Calculations show that with increase of Lu the mean dimensionless transfer potentials undergo a marked change: the temperature of the material drops, while the mean value of mass content and of filtration potential increases. The changes of the latter are particularly marked with increase of the Luikov criterion. Thus, we have at Lu $= 0{\cdot}3$ $\bar{P} = 0{\cdot}7268$ and, respectively, at Lu $= 0{\cdot}7$ $\bar{P} = 60{\cdot}57$, but at Lu $= 1{\cdot}0$ $\bar{P} = 83{\cdot}95$.

Another criterion for inertia or correlation between heat and mass transfer is $\mathrm{Lu}_p = a_p/a_q$. It characterizes the inter-connexion between molar transfer of matter within the material, and the transfer of heat; in other words, it characterizes the inertia of the fields of filtration potential (fields of excess pressure) with respect to temperature fields. It can be seen from Fig. 4 that criterion Lu_p is automodelous with respect to $\bar{T}$ and $\bar{\Theta}$; it affects only the pressure field of the material. In connexion with the dependence of heat and mass transfer on molar similarity criteria let us consider in greater detail the dynamics of change of filtration potential.

During intensive heating of moist dispersed media very strong vapour formation occurs inside the material. This leads to the formation of a stable gradient of filtration potential—the gradient of general or excess pressure. The latter is determined by the commensurability of the time of relaxation of pressure through the skeleton of the material, and the formation within the same period of time of vapour necessary for the restitution of the original state. The distribution of filtration potential depends on the method of heating the medium and also on the values of the thermophysical and mechanical

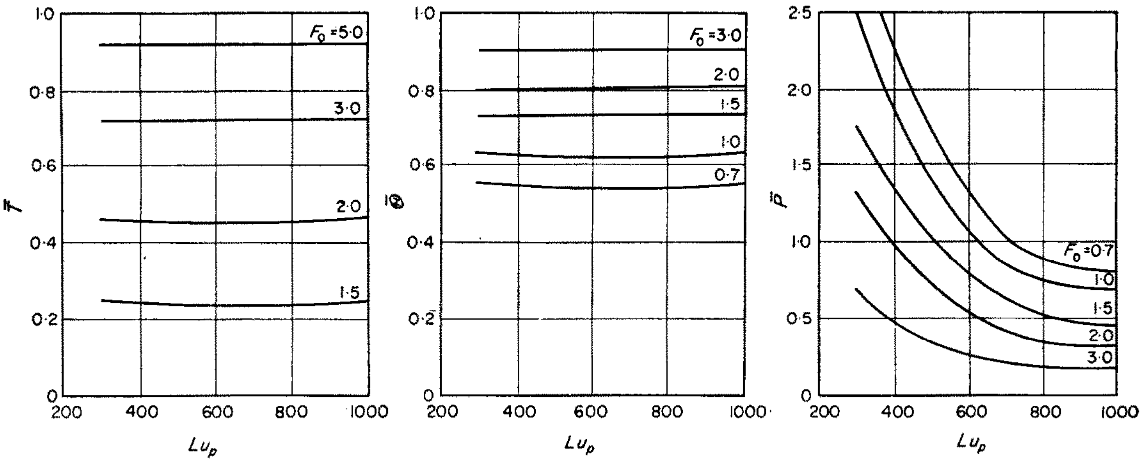

FIG. 4. Dimensionless transfer potentials as a function of Lu_p criterion

characteristics of the material. However, qualitatively speaking, in any case there is a strong increase of the potential at the beginning stage of the process. With decrease of mass content (i.e. with increase of $\bar{\Theta}$) the intensity of internal vapour formation diminishes. That, in its turn, leads to a decrease of the gradient of filtration potential. The results of analysis of the solutions of the system of equations (1) are in complete accordance with experimental results [8].

Considering the effect of various criteria on processes of heat and mass exchange simplified criterial equations can be suggested describing internal heat and mass transfer. The equations are:

$$T = T\,(\mathrm{Lu}, \mathrm{Bi}_q, \mathrm{Pn}, \mathrm{Ko}, \epsilon, \mathrm{Fo}, x/R)$$

$$\Theta = \Theta\,(\mathrm{Lu}, \mathrm{Bi}_m, \mathrm{Pn}, \mathrm{Ko}, \epsilon, \mathrm{Fo}, x/R)$$

$$P = P\,(\mathrm{Lu}, \mathrm{Lu}_p, \mathrm{Bu}, \mathrm{Pn}, \mathrm{Ko}, \epsilon, \mathrm{Fo}, x/R)$$

A further analysis of highly intensive heat and mass transfer will most probably permit still further simplification the criterial equations by finding correlations between the separate similarity criteria in the above-mentioned expressions. There is reason to believe [6] that in the temperature expression Bi_q and Ko appear in form of the ratio $\mathrm{Bi}_q/\mathrm{Ko}$, and, similarly, in the expression for the transfer potential Bi_m and Pn appear in form of the ratio $\mathrm{Bi}_m/\mathrm{Pn}$.

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